Differential Hall-Effect Spectroscopy (DHES) for p-type semiconductor

The hole concentration $p(T)$ produced by n different acceptor species (density $N_{\mathrm{A}i}$ and energy level $E_{\mathrm{A}i}$) and one donor (density N_{D}) is expressed by

$$p(T)=\sum_{i=1}^{n}\frac{N_{\mathrm{A}i}}{1+g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)}-N_{\mathrm{A}}\ . \tag{1}$$

The derivative $\mathrm{kT}\cdot \mathrm{d}p(T)/\mathrm{d}E_{\mathrm{F}}$ is derived as

$$kT\frac{dp(T)}{dE_{\mathrm{F}}}=-kT\sum_{i=1}^{n}N_{\mathrm{A}i}\frac{\frac{\partial}{\partial E_{\mathrm{F}}}\left[1+g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)\right]+\frac{\partial}{\partial(kT)}\left[1+g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)\right]\frac{\partial(kT)}{\partial E_{\mathrm{F}}}}{\left[1+g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)\right]^{2}}$$

$$=kT\sum_{i=1}^{n}N_{\mathrm{A}i}\frac{\frac{g}{kT}\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)+g\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{(kT)^{2}}\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)\frac{\partial(kT)}{\partial E_{\mathrm{F}}}}{\left[1+g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)\right]^{2}}$$

$$=\sum_{i=1}^{n}N_{\mathrm{A}i}\frac{g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)}{\left[1+g\exp\left(\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\right)\right]^{2}}\cdot\left[1+\frac{E_{\mathrm{A}i}-E_{\mathrm{F}}}{kT}\cdot\frac{\partial(kT)}{\partial E_{\mathrm{F}}}\right] \tag{2}$$

Since energy levels measured from the top of the valence band are described as

$$\Delta E_{\mathrm{A}i}=E_{\mathrm{A}i}-E_{\mathrm{V}} \tag{3}$$

and

$$\Delta E_{\mathrm{F}}=E_{\mathrm{F}}-E_{\mathrm{V}}\ , \tag{4}$$

the DHES signal is theoretically expressed by

$$DHES[\Delta E_{\mathrm{F}}(T)]=\sum_{i=1}^{n}N_{\mathrm{A}i}\frac{g\exp\left(\frac{\Delta E_{\mathrm{A}i}-\Delta E_{\mathrm{F}}}{kT}\right)}{\left[1+g\exp\left(\frac{\Delta E_{\mathrm{A}i}-\Delta E_{\mathrm{F}}}{kT}\right)\right]^{2}}\cdot\left[1+\left(\frac{\Delta E_{\mathrm{A}i}-\Delta E_{\mathrm{F}}}{kT}\right)\cdot\frac{\partial(kT)}{\partial\Delta E_{\mathrm{F}}}\right]. \tag{5}$$

The function

$$N_{\mathrm{A}i}\frac{g\exp\left(\frac{\Delta E_{\mathrm{A}i}-\Delta E_{\mathrm{F}}}{kT}\right)}{\left[1+g\exp\left(\frac{\Delta E_{\mathrm{A}i}-\Delta E_{\mathrm{F}}}{kT}\right)\right]^{2}}$$

has a maximum of

$$\frac{N_{\mathrm{A}i}}{4}$$

at $\Delta E_{\mathrm{F}}=\Delta E_{\mathrm{A}i}+kT_{\mathrm{max}}\ln g$.