Differential Hall-Effect Spectroscopy (DHES) for n-type semiconductor

The electron concentration $n(T)$ produced by n different donor species (density $N_{\mathrm{D}i}$ and energy level $E_{\mathrm{D}i}$) and one acceptor (density N_{A}) is expressed by

$$n(T) = \sum_{i=1}^{n} \frac{N_{\mathrm{D}i}}{1 + g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)} - N_{\mathrm{A}} . \tag{1}$$

The derivative $(\text{-kT})\mathrm{d}n(T)/\mathrm{d}E_{\mathrm{F}}$ is derived as

$$\begin{aligned}
(-kT)\frac{dn(T)}{dE_{\mathrm{F}}} &= kT\sum_{i=1}^{n} N_{\mathrm{D}i} \frac{\dfrac{\partial}{\partial E_{\mathrm{F}}}\left[1 + g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)\right] + \dfrac{\partial}{\partial (kT)}\left[1 + g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)\right]\dfrac{\partial (kT)}{\partial E_{\mathrm{F}}}}{\left[1 + g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)\right]^2} \\
&= kT\sum_{i=1}^{n} N_{\mathrm{D}i} \frac{\dfrac{g}{kT}\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right) + g\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{(kT)^2}\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)\dfrac{\partial (kT)}{\partial E_{\mathrm{F}}}}{\left[1 + g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)\right]^2} \\
&= \sum_{i=1}^{n} N_{\mathrm{D}i} \frac{g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)}{\left[1 + g\exp\left(\dfrac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT}\right)\right]^2} \cdot \left[1 - \frac{E_{\mathrm{F}} - E_{\mathrm{D}i}}{kT} \cdot \frac{\partial (kT)}{\partial E_{\mathrm{F}}}\right]
\end{aligned} \tag{2}$$

Since energy levels measured from the bottom of the conduction band are described as

$$\Delta E_{\mathrm{D}i} = E_{\mathrm{C}} - E_{\mathrm{D}i} \tag{3}$$

and

$$\Delta E_{\mathrm{F}} = E_{\mathrm{C}} - E_{\mathrm{F}} , \tag{4}$$

the DHES signal is theoretically expressed by

$$DHES\left[\Delta E_{\mathrm{F}}(T)\right] = \sum_{i=1}^{n} N_{\mathrm{D}i} \frac{g\exp\left(\dfrac{\Delta E_{\mathrm{D}i} - \Delta E_{\mathrm{F}}}{kT}\right)}{\left[1 + g\exp\left(\dfrac{\Delta E_{\mathrm{D}i} - \Delta E_{\mathrm{F}}}{kT}\right)\right]^2} \cdot \left[1 + \left(\frac{\Delta E_{\mathrm{D}i} - \Delta E_{\mathrm{F}}}{kT}\right) \cdot \frac{\partial (kT)}{\partial \Delta E_{\mathrm{F}}}\right] . \tag{5}$$

The function

$$N_{\mathrm{D}i} \frac{g\exp\left(\dfrac{\Delta E_{\mathrm{D}i} - \Delta E_{\mathrm{F}}}{kT}\right)}{\left[1 + g\exp\left(\dfrac{\Delta E_{\mathrm{D}i} - \Delta E_{\mathrm{F}}}{kT}\right)\right]^2}$$

has a maximum of

$$\frac{N_{\mathrm{D}i}}{4}$$

at $\Delta E_{\mathrm{F}} = \Delta E_{\mathrm{D}i} + kT_{\max} \ln g$.